When Noise Accumulates

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Abstract

We propose a model for noisy quantum evolutions where the noise is forced to accumulate, and consider related noise models, called "detrimental noise," that will cause quantum error correction and fault-tolerant quantum computation to fail. We start with properties of detrimental noise for two qubits and proceed to a discussion of highly entangled states, the rate of noise, and general noisy quantum systems.

1 Introduction

Quantum computers were offered by Feynman and others and formally described by Deutsch [13]. The idea was that since computations in quantum physics require an exponential number of steps on digital computers, computers based on quantum physics may outperform classical computers. A spectacular support for this idea came with Shor's theorem [28] that asserts that factoring is in BQP (the complexity class described by efficient quantum computation).

The feasibility of computationally superior quantum computers is one of the most fascinating and clear-cut scientific problems of our time. The main concern regarding quantum-computer feasibility is that quantum systems are inherently noisy. This concern was put forward in the mid-90s by Landauer [22, 23], Unruh [31], and others. The theory of quantum error correction and fault-tolerant quantum computation (FTQC) and, in particular, the *threshold theorem* [2, 18, 19], which asserts that under certain conditions FTQC is possible, provides strong support for the possibility of building quantum computers. However, as far as we know, quantum error correction and quantum fault tolerance (and the very highly entangled quantum states that enable them) are not experienced in natural quantum processes. It is therefore not clear if computationally superior quantum computation is necessary to describe natural quantum processes.

We will try to address two closely related questions. The first and main one is, what are the properties of quantum processes that do not exhibit quantum fault tolerance and how are such processes formally modelled. The second is, what kind of noise models cause quantum error correction and FTQC to fail.

A main point we would like to make is that it is possible that there is a systematic relation between the noise and the intended state of a quantum computer. Such a systematic relation does not violate linearity of quantum mechanics, and it is expected to occur in processes that do not exhibit fault tolerance. Let me give an example: suppose that we want to simulate on a noisy quantum computer a certain bosonic state. The standard view of noisy quantum computers asserts that this can be done up to some error which strongly depends on the computational basis. In contrast, the type of noise we expect to express noise accumulation amounts to having a mixed state between the intended bosonic state and other bosonic states (that represent the noise). Such a noise does not exhibit a strong dependence on the computational basis but rather it depends on the intrinsic properties of the

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simulated state. More generally, the hope regarding FTQC is that no matter what the quantum computer computes or simulates, nearly all of the noise will be a mixture of states that are not codewords in the error correcting code, but which are correctable to states in the code. Our point of view suggests that the process for creating a quantum error correcting code will necessarily lead to a mixture of the desired codeword with undesired codewords.

We consider two levels of abstraction. For quantum computers and for many other quantum systems there is a natural tensor product structure on the Hilbert space describing the state of the system. This allows us to talk about notions like locality and entanglement. The more abstract setting is based on states described by a general Hilbert space without any additional structure. Here, we cannot talk about entanglement and locality but nevertheless we expect that insight about noise accumulation and related noise models should extend to this general level of abstraction.

We will now describe the structure of the paper. While a major property of FTQC is that it allows suppression of noise propagation, in Section 2 we propose a mathematical model that aims to describe quantum evolutions with un-suppressed noise propagation. This model is described in the abstract setting where no structure on the Hilbert space of states is assumed. The class of noisy quantum processes we describe is an interesting *subclass* of the class of all noisy quantum processes described by Lindblad equations. The model is based on the standard Lindblad equation with a certain additional "smoothing" in time. A formal definition of detrimental noise based on this model is given. Smoothed Lindblad evolutions are further discussed in Section 7 in the Appendix.

The next three sections discuss the restricted model of noisy quantum computers. (Section 8 in the Appendix describes the basic framework for noisy quantum computers and the threshold theorem.) In Section 3 we discuss highly correlated noise, the notion of noise synchronization, and the rate of highly correlated noise. In Section 4, we propose two conjectures about detrimental noise: the first is in terms of two-qubit behavior, and the second is in terms of many highly entangled qubits states. The two-qubit conjecture asserts informally that information leaks for two entangled qubits are necessarily positively correlated. The second conjecture asserts that the noise for a highly entangled state manifests strong error synchronization. In Section 5 we ask what kind of states are possible for noisy quantum computers that satisfy our conjectures, and what kind of highly entangled states are not supported by such a model. In Section 6 we discuss some computational complexity aspects and some physical aspects of our point of view.

2 Modeling quantum systems with un-suppressed noise propagation

A main property of FTQC is that it enables us to suppress noise propagation: the effect of the noise at a certain computer cycle diminishes almost completely already after a constant number of computer cycles. In this section we would like to formally model quantum systems for which noise propagation is not suppressed.

A way to force un-suppressed noise propagation into the model is as follows. Start with an ideal unitary quantum evolution $\rho_t : 0 \leq t \leq 1$ on some Hilbert space $\mathcal{H}$. Suppose that $U_{s,t}$ denotes the unitary operator describing the transformation from time s to time t, ($s < t$). ρ_t is thus described by the abstract Schrodinger equation

$$d\rho/dt = -i[H_t, \rho]. \tag{1}$$

Next consider a noisy version where E_t is a superoperator describing the infinitesimal noise at time t. This data allows us to describe the noisy evolution σ_t via the Lindblad equation

$$d\sigma/dt = -i[H_t, \sigma] + E_t(\sigma). \tag{2}$$

We will now describe a certain "smoothing" in time of the noise. Let K be a positive continuous function on [-1,1]. We write $\bar{K}(t) = \int_{t-1}^{t} K(s)ds$. Replace the noise superoperator E_t at time t by

$$\tilde{E}_t = (1/\bar{K}(t)) \cdot \int_0^1 K(t-s) U_{s,t} E_s U_{s,t}^{-1} ds. \tag{3}$$

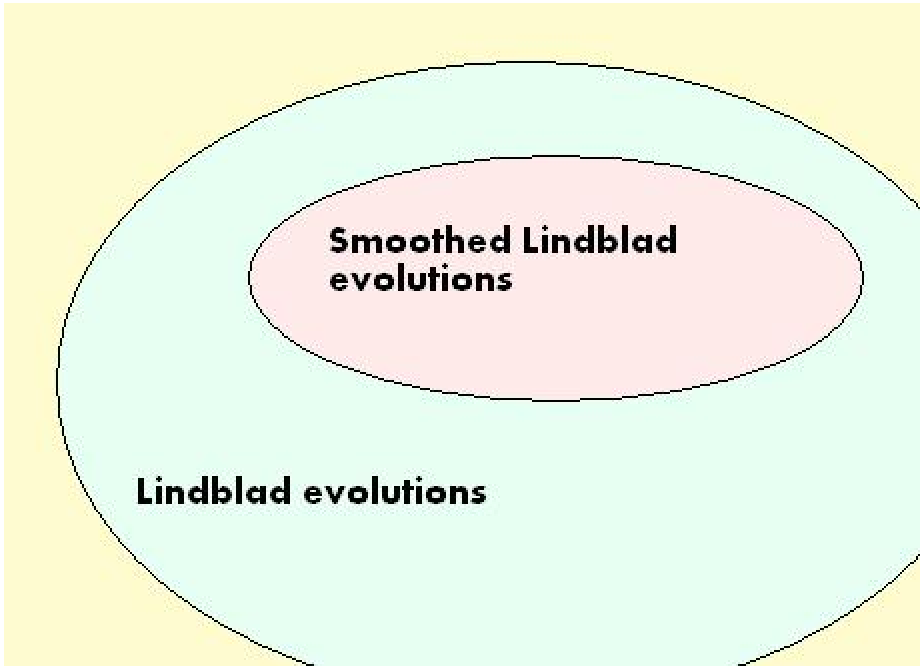

Figure 1. Smoothed Lindblad evolutions are a restricted subclass of the class of all Lindblad evolutions

We denote by $\tilde{\sigma}_t$ the noisy evolution described by the smoothed noise superoperator

$$d\tilde{\sigma}/dt = -i[H_t, \tilde{\sigma}] + \tilde{E}_t(\tilde{\sigma}). \tag{4}$$

We refer to such evolutions as *smoothed Lindblad evolutions*.

For the rest of the paper we will restrict somewhat the class of noise superoperators and we will suppose that E_t and hence E'_t are described by POVM-measurements (see [25], Chapter 2).

> **Definition:** *Detrimental noise* refers to noise (described by a POVM-measurement) that can be described by equation (3).
>
> **Main Conjecture:** Noisy quantum processes are subject to detrimental noise.

Relation (3) is offered as a mathematical device to describe the situation where noise propagation is not suppressed. Regarding $\tilde{E}_t$ rather than E_t as representing the noise at time t reflects a different way of "bookkeeping" the noise which will make almost no difference to systems unless they exhibit the massive cancellation of noise achieved through quantum error correction and quantum fault tolerance. Relation (3) can represent various scenarios. It may apply to noisy quantum circuits with standard noise above the threshold. Alternatively, it may apply simply to standard noisy quantum circuits that do not contain error-correction ingredients. Relation (3) is a proposed mathematical description of quantum processes that do not enact quantum fault tolerance, or alternatively, a desciption of a type of noise which resists quantum fault tolerance. See Section 7 for rurther discussion.

We can replace relation (3) by a discrete-time description. When we consider a quantum computer that runs for T computer cycles, we start with standard storage noise E_t for the t-step. Then we consider the noise operator

$$E'_t = 1/(\sum_{s=1}^{T} K((t-s)/T)) \cdot \sum_{s=1}^{T} K((t-s)/T) U_{s,t} E_t U_{s,t}^{-1}, \tag{5}$$

where again $U_{s,t}$ is the intended unitary operation between step s and step t.

3 Correlated noise and noise synchronization

3.1 Describing error synchronization via Pauli expansion

In this section we restrict the discussion to models of noisy quantum computers and discuss highly correlated errors. For background regarding noisy quantum computers and a statement of the "threshold theorem" the reader is referred to Section 8. Highley correlated errors are further discussed in Section 9. The concern regarding highly correlated noise has been raised in several papers [26, 5], yet there have been only a few systematic attempts to study what kind of correlated errors will cause the threshold theorem to fail.

Error synchronization refers to a situation where, while the expected number of qubit errors is small, there is a substantial probability of errors affecting a large fraction of qubits. A simple way to describe error synchronization is via the expansion of the quantum operation E in terms of multi-Pauli operators. A quantum operation E can be expressed as a linear combination

$$E = \sum v^w P_w, \tag{6}$$

where w is a word of length n $(i_1, i_2, \ldots, i_n)$, and $i_k \in \{I, X, Y, Z\}$ for every k, v^w is a vector, and P^w is the quantum operation that corresponds to the tensor product of Pauli operators whose action on the individual qubits is described by the multi-index w.

The amount of error on the kth qubit is described by $\sum\{\|v^w\|_2^2 : i_k \neq I\}$. For a multi-index w define $|w| := |\{k : i_k \neq I\}|$. Let

$$f(s) := \sum\{\|v^w\|_2^2 : |w| = s\}.$$

We regard $\sum_{s=1}^{n} f(s)s$ as the *expected number of qubit errors*.

Define the *rich error syndrome* to be the probability distribution described by assigning to the word w the value $\|v^w\|$ (normalized). We will define the *coarse error syndrome* as the binary word of length n obtained from w by replacing I with '0' and the other letters by '1'. Given a noise operation E, the distribution $\mathcal{E}$ of the rich error syndrome is an important feature of the noise. Given E we will denote by $\mathcal{D}$ the probability distribution of coarse error syndrome. $f(s)$ is simply the probability of a word drawn according $\mathcal{D}$ having s '1's.

Suppose that the expected number of qubit errors is αn where n is the number of qubits. All noise models studied in the original papers of the threshold theorem, as well as some extensions that allow time- and space-dependencies (e.g., [30, 7, 3]), have the property that $f(s)$ decays exponentially (with n) for $s = (\alpha + \epsilon)n$, where $\epsilon > 0$ is any fixed real number. (This is particularly simple when we consider storage error, which is statistically independent over different qubits.) In contrast, we say that E leads to *error synchronization* if $f(\geq s)$ is substantial for some $s \gg \alpha n$. We say that E leads to a *very strong* error synchronization if $f(\geq s)$ is substantial for $s = 3/4 - \delta$ where $\delta = o(1)$ as n tends to infinity. By "substantial" we mean larger than some absolute constant times α/s, or, in other words, the multi-Pauli terms for $|I| \geq s$ contributes a constant fraction of the expected number of qubit errors.

Remark: Error syndromes obtained by measuring the noise in terms of the tensor product of Pauli operators is an important ingredient of several fault-tolerant schemes. Note that our definition of the rich error syndrome (unlike error-syndromes used in quantum error correction) is based on the quantum operation E representing the noise. (Since quantum states can have non-trivial Pauli stabilizers the rich error syndrome is not defined uniquely just in terms of the intended and noisy states.)

3.2 The rate of highly correlated noise

Recall that the trace distance $D(\sigma, \rho)$ between two density matrices ρ and σ is equal to the maximum difference in the results of measuring ρ and σ in the same basis. $D(\sigma, \rho) = 1/2\|\sigma - \rho\|_{tr}$. When the error is represented by a quantum operation E the rate of error for an individual qubit is the maximum over all possible states ρ of the qubit of the trace distance between ρ and $E(\rho)$.

Highly correlated errors are damaging for quantum error correction, but a potentially even more damaging property we face for highly correlated noise is that the notion of "rate of noise for individual qubits" becomes sharply different from the rate of noise as measured by trace distance for the entire Hilbert space describing the state of the computer.

Consider two extreme scenarios. In the first scenario, for a time interval of length t there is a depolarizing storage noise that hits every qubit with probability pt. In the second scenario the noise is highly correlated: all qubits are hit with probability pt and with probability $(1 - pt)$ nothing happens. In terms of the expected number of qubit errors both these noises represent the same rate. The probability of every qubit being corrupted at a time interval of length t is pt. However, in terms of trace distance (and here we must assume that t is very small), the rate of the correlated noise is n^{-1} times that of the uncorrelated noise. What should be the correct assumption for the rate of noise when we move away from the statistical independence assumption? If noise propagation is the "role model" then measuring the noise in terms of trace distance for the entire Hilbert space appears to be correct.

4 Detrimental noise from two qubits to many

4.1 Two conjectures

In this subsection we present qualitative statements of two conjectures concerning decoherence for quantum computers which, if (or when) true, are damaging to quantum error correction and fault tolerance. The first conjecture concerns entangled pairs of qubits.

> **Conjecture A:** A noisy quantum computer is subject to error with the property that information leaks for two substantially entangled qubits have a substantial positive correlation.

We emphasize that Conjecture A refers to part of the overall error affecting a noisy quantum computer. Other forms of errors and, in particular, errors consistent with current noise models may also be present. We conjecture that detrimental noise described by smoothed Lindblad evolutions obtained by applying relation (3) on standard noise for quantum computers will satisfy Conjectures A and B, but our study of these conjectures is so far separated from the more abstract definition of detrimental noise.

Recall that error synchronization refers to a situation where, although the error rate is small, there is nevertheless a substantial probability that errors will affect a large fraction of qubits.

> **Conjecture B:** In any noisy quantum computer in a highly entangled state there will be a strong effect of error synchronization.

We should informally explain already at this point why these conjectures, if true, are damaging. We start with Conjecture B. The states of quantum computers that apply error-correcting codes needed for FTQC are highly entangled (by any formal definition of "high entanglement"). Conjecture B will imply that at every computer cycle there will be a small but substantial probability that the number of faulty qubits will be much larger than the threshold.[1] This is in contrast to standard assumptions that the probability of the number of faulty qubits being much larger than the threshold decreases exponentially with the number of qubits. Having a small but substantial probability of a large number of qubits being faulty is enough to cause the quantum error-correction codes to fail.

Why is conjecture A damaging? Here the situation is trickier since without some additional assumptions conjecture A is not relevant to the highly entangles states used for FTQC. For such states, pairs of qubits are not

[1] Here we continue to assume that the probability of a qubit being faulty is small for every computer cycle.

entangled. Let us make the additional assumption that individual qubits can be measured without inducing errors on other qubits. This is a standard assumption regarding noisy quantum computers.[2] When we start from highly entangled states needed for FTQC and measure (and look at the results for) all but two qubits, we will reach pairs of qubits (whose intended state is pure) with almost statistically independent noise, in contrast to Conjecture A. Under this assumption it is also possible to deduce Conjecture B from Conjecture A.

4.2 Mathematical formulation of Conjecture A

In this subsection we will describe a mathematical formulation of Conjecture A. The first step in this formal definition is to restrict our attention to noise described by POVM-measurements. This is a large class of quantum operations describing information leaks from the quantum computer to the environment.

Our setting is as follows. Let ρ be the intended ("ideal") state of the computer and consider two qubits a and b. Consider a POVM-measurement E representing the noise. We describe correlation between the qubit errors via the expansion in tensor products of Pauli operators, or, in other words, via the error syndrome. Associated to E (see Section 3.1) is a distribution $\mathcal{E}(E)$ of error syndromes, i.e., words of length n in the alphabet $\{I, X, Y, Z\}$. A coarser distribution $\mathcal{D}(E)$ of binary strings of length n is obtained by replacing the letter I with '0' and all other letters by '1'. As a measure of correlation $cor_{i,j}(E)$ between information leaks for the ith and jth qubit we will simply take the correlation between the events $x_i = 1$ and $x_j = 1$ according to $\mathcal{D}(E)$. We also define $r_i(E)$ as the probability that $x_i = 1$ according to the distribution $\mathcal{D}$.

We now discuss how to measure entanglement. Suppose that ρ is the intended state of the computer. For a set Z of qubits and a state ρ we denote by $\rho|_Z$ the density matrix obtained after tracing out the qubits not in Z. If Z contains only the ith qubit, we write ρ_i instead of $\rho|_Z$. As a measure of entanglement we simply take the trace distance between the state induced on the two qubits and a separable state. Formally, let $SEP(i, j)$ denote the set of mixed separable states on $Z = \{i\} \cup \{j\}$, namely, states that are mixtures of tensor product pure states $\tau = \tau_i \otimes \tau_j$. Define $Ent(\rho : i, j) = max\{\|\rho_{i,j} - \psi\| : \psi \in SEP(i, j)\}$.

Here is the statement of Conjecture A for two qubits:

Conjecture A: (mathematical formulation)

For every two qubits

$$cor_{i,j}(E) \geq K(r_i(E), r_j(E)) \cdot Ent(\rho : i, j). \tag{7}$$

Here, $K(x, y)$ is a function of x and y so that $K(x, y)/\min(x, y)^2 \gg 1$ when x and y are positive and small. (Note that Conjecture A) does not claim anything when the two qubits are noiseless.) If $r_i(E) = r_j(E) = \alpha$ for a small real number α, then the conjecture asserts that $cor_{i,j}(E) \gg \alpha^2$, and, as we will see later, this is what is needed to derive error synchronization.

Remark: We mainly use Conjecture A for the case where the two qubits are in joint pure state. In this case we can simply take the entropy of one of the qubits as the measure of entanglement.

4.3 Emergent entanglement and Conjecture B

We now describe Conjecture B formally and propose a strong form of Conjecture A for two qubits based on a notion of "emergent entanglement."

Definition: The *emergent entanglement* of two qubits is the maximum over all *separable* measurements of the remaining qubits of the expected amount of entanglement between the two qubits when we look at the outcome of the measurements.

[2]It should be emphasized that the assumption that we can *always* measure a qubit without inducing errors on others, goes contrary to the picture of noisy quantum computers we try to draw. We use it to examine stronger forms of the notion of entanglement that are relevant.

Define a *highly entangled state* as a state where the expected emergent entanglement among pairs of qubits is large. This is the case for states used in quantum error correction. A strong form of Conjecture A is obtained if we take emergent entanglement as the measure of entanglement.

Theorem 4.1. *For noisy quantum computers Conjecture A implies conjecture B in the following two cases:*
(1) When we add the assumptions that qubits can be measured without introducing noise on other qubits.
(2) When we formulate Conjecture A for "emergent entanglement".

The proof is based on applying Proposition 4.2 below to the coarse error-syndrome.

Proposition 4.2. *Let $\eta < 1/20$ and $s > 4\eta$. Suppose that $\mathcal{D}$ is a distribution of 0-1 strings of length n such that $p_i(\mathcal{D}) \geq \eta$ and $cor_{ij}(\mathcal{D}) \geq s$. Then*

$$\mathbf{Prob}(\sum_{i=1}^{n} x_i > sn/2) > s\eta/4. \tag{8}$$

It will be interesting to relate Conjectures A and B to smoothed Lindblad evolutions and to the main conjecture of Section 2. It will also be interesting to check whether the assertion of Conjectures A and B holds for noisy adiabatic computers [12], and for various other models of noisy quantum computations. Several extensions of Conjecture A to pairs of qudits (rather than qubits), and to a larger number of qubits are proposed in [15, 14]. Several alternative approaches for how to measure the correlation between "information leaks" for Conjecture A and how to define "highly entangled states" for Conjecture B are also considered. Finally, it is worth noting that our interpretation to Conjectures A and B is slightly different. Conjecture B is a physics conjecture asserting that for highly entangles states strong effects of noise synchronization cannot be avoided. Conjecture A should be regarded as a conjectured requirement on appropriate modeling of noisy quantum computers.

5 Censorship and testable implications

We can expect that detrimental noise will lead to "very highly entangled states" being completely infeasible for noisy quantum computers. Limitations on feasible states of a quantum computer are referred to as "censorship." Computational complexity imposes severe restrictions on the feasible states of (noiseless) quantum computers. For example, a state that is approximately the outcome of a random unitary operator on the entire 2^n-dimensional Hilbert space is computationally out of reach when the number of qubits is large. States of noisy quantum computers according to the standard noise model have, of course, further restrictions (e.g., they are never pure), but the hypothesis of fault tolerant quantum computing asserts that when we move from the physical qubits to certain "logical" qubits, the only restriction on feasible quantum states is computational. Moreover, quantum fault tolerance can lead (after discarding a large number of warm qubits) to physical qubits whose joint state is close to being pure and is highly entangled.

We expect that detrimental noise will lead to further restrictions of a statistical nature on feasible states for noisy quantum computers and following is a concrete conjecture in this direction. Given a set B of m qubits, consider the convex hull F of all states that factor into a state on k of the qubits tensor a state on the other $m-k$ qubits, for all k. For a state ρ on B define $Ent(\rho; B)$ as the trace distance between $\rho|_B$ and F. (Alternatively, we can substitute the minimum relative entropy for the trace distance.) Now define, $Ent(\rho; B)$ as the trace distance between ρ and F. Next define

$$\widetilde{ENT}(\rho) = \sum_B \{Ent(\rho; B) : B \subset A\}. \tag{9}$$

Conjecture C: There is a polynomial P (perhaps even a quadratic polynomial) such that for any quantum computer on n qubits, which describes a state ρ (which need not be pure),

$$\widetilde{ENT}(\rho) \leq P(n). \tag{10}$$

Suppose that we are given a noisy quantum computer where separable pure states can be achieved up to a certain small error η. For noisy quantum computers under the standard model of noise, every pure state that is computationally feasible can be approximated (after discarding a large number of qubits) up to an error that is a small constant times η. Conjecture C asserts that some states that are computationally feasible cannot be approximated at all and thus they do not correspond to any quantum states we may expect in nature.

Perhaps most interesting are states that can be achieved under our assumption of accumulated noise and thus may represent familiar physics phenomena. One interesting potential aspect of statistical censorship (following suggestions by Ronnie Kosloff) is that there are mixed states that cannot be "cooled." Namely, certain types of entangled states can only occur in "high enough" entropy. Such a property is not expected for computationally based censorship.

In our models the quality of approximation depends on the state. The noise level (or minimum entropy we can achieve) for separable states translates to a higher noise level (minimum entropy) for more complicated entangled states. This relies on the assumption that all operations including cooling itself are local, in addition to the assumption that noise accumulates.

6 Discussion of computational complexity and physical aspects

We will briefly discuss some computational complexity aspects of our conjectures. Noise accumulation appears to imply not only that the errors will be correlated for very entangled states but also that the error rate in terms of the expected number of qubit errors in one computer cycle will scale up. (This is due to the discrepancy between measuring error in terms of trace distance and in terms of qubit errors which occurs when the errors are highly correlated.) It will be interesting to show that imposing noise accumulation reduces the computational power of noisy quantum computers to BPP. This may have applications for various suggestions for simple quantum devices that might already exhibit superior computational power, and for the ability to simulate on classical computers various specific quantum processes occurring in nature.

We now turn our attention to some physical aspects of the conjectures. One place to examine some suggestions of this paper is current implementations of ion-trap computers. In these implementations we need to move qubits together in order to gate them, and this suggests that, in each computer cycle, errors will be correlated for *all* pairs of qubits. At present, the rate of noise is still the major concern of experimentalists, but it is not clear how a large pairwise correlation between all pairs of qubits can be avoided in the current architecture. This is an example, where properties of accumulated noise occur for other reasons. We note, however, that experimental advances for *single*-qubit ion trap evolutions are in tension with certain strong forms of our conjectures (see Section 7).

Let us go back to the example of simulating bosonic states with a noisy quantum computer. When errors accumulate we expect that a large (even dominant) part of the noise will not consist of local noise based on the computational bases but rather it will be a mix of the intended bosonic state with other unintended bosonic states. Noise accumulation seems consistent with the familiar property of physical systems where the low-scale structure is not witnessed when we look at larger scales. We do not yet have quantum computers that simulate bosonic states but we do have several natural and experimental processes that come close to this description, like phonons, which can be regarded as a bosonic state (on a macroscopic scale) "simulated" on microscopic "qudits". Another relevant example is that of Bose-Einstein condensation on cold atoms. Describing the bosonic state in terms of individual atoms is analogous to describing a complicated state of a quantum computer in terms of the computational basis. This analogy enables us to ask if the deviation of a state created experimentally from a pure state can be described

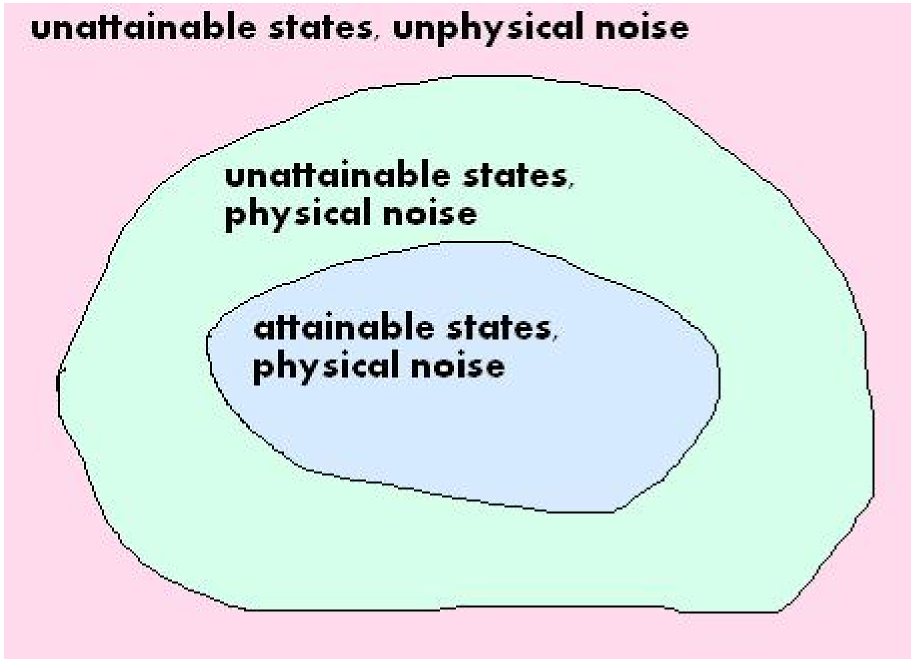

Figure 2. Given a proposed architecture for a quantum computer it is possible that for some hypothetical states that cannot be achieved the proposed properties of noise are "unphysical." The place to examine the conjectures is for attainable states.

by independent noise operators on the different atoms. We propose a different picture, namely, that a state created experimentally can be described as a mixture of different pure Bose-Einstein states and that, with respect to any pure state it approximates, it exhibits strong error synchronizations on the "atomic basis" for the underlying Hilbert space. These examples can serve as a good place to examine noise.

Several people have commented that our suggested properties of noise for some (hypothetical) quantum computer architecture at some quantum state ρ allow instantaneous signaling, and thus violate basic physical principles. This is perfectly correct, but our proposed conclusion is that this quantum computer architecture simply does not accommodate the quantum state ρ. (See Figure 2.)

Finally, another comment was that FTQC via topological quantum computing does not rely on the threshold theorem and Conjectures A and B are not relevant for this model. However, the mathematics underlying both the threshold theorem and FTQC via topological quantum computers is quite similar. The extreme stability to noise expected for anyonic systems relies on similar assumptions to those enabling quantum error correction. Creating anyonic systems that may allow FTQC via topological quantum computing already requires fault-tolerant processes. When we create Abelian anyons in the laboratory, or try to create non-Abelian anyons, there is no reason to believe that the process for creating them will involve suppression of propagated noise. I conjecture that smoothed Lindblad evolutions describe the current and proposed processes for creating anyons, and that just as when we simulate fermions or bosons, we will witness a mixture of the intended state with other states of the same type and we will not witness the strong stability of certain anyonic systems that is predicted by current models. Of course, these matters are subject to intense experimental examinations.

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APPENDIX

7 The rate of noise, single qubit ion trap computers, and drunken evolutions

This section complements Section 2 in its treatment of general quantum noisy systems. We would like to discuss the following questions:

1. What can be assumed about the rate of noise?
2. Can all noisy natural evolutions be described by smoothed Lindblad evolutions?
3. To what extent are our suggestions compatible with the experimental behavior of single-qubit ion traps?
4. Can smoothed Lindblad evolutions be the basis for a model of noisy quantum computers?
5. Can the idea of smoothing-in-time be further abstracted without a reference to noise or with noise being an emergent phenomenon?

7.1 The rate of noise

A coherent picture of noisy quantum evolutions that do not demonstrate fault tolerance needs to say something about the rate of noise. The following lower bound for the rate of noise in quantum evolutions is inspired by papers of Boxio, Viola, and Ortiz (see [10]) on canonical coherent state.[3]

[3] I am thankful to Michael Khasin who brought the work of Boxio, Viola, and Ortiz to my attention.

Conjecture E: A noisy quantum system is subject to (detrimental) noise with the following property: the rate of noise at a time-interval $[s, t]$ (in terms of trace distance) is bounded from below by a measure of noncommutativity for the set of operators describing the evolution in this interval.

Since we will not use any quantitative version of this conjecture we will not attempt to give a precise mathematical form, but only describe how to express it mathematically in rough terms. Given an algebra of operators we consider the set of projections (corresponding to measurements) in the family. If all these projections commute then the measure of non-commutativity is 0. Otherwise, the measure of noncommutativity is based on norms of commutators of projections in the family.

The hope is to find an appropriate version of Conjecture E that together with the assumption that the noisy quantum process is described by smoothed Lindblad evolutions will imply that the assertion of Conjecture E is inherited to subsystems of the original system and to systems described by subspaces of the underlying Hilbert space.

7.2 Describing noisy processes with smoothed Lindblad evolutions

A stronger form of the main conjecture from Section 2 is that noisy quantum evolutions in nature can always be described by smoothed Lindblad evolutions. This can be expected if we cannot witness quantum fault tolerance and if the proposed smoothing-in-time is the appropriate way to express quantum evolutions without fault tolerance.

One concern with this suggestion is that the class of Lindblad evolutions is not general enough to describe the most general types of noise. This can possibly be dealt with by applying our smoothing-in-time for more general forms of quantum evolutions. Another concern that seems more damaging was raised by Greg Kuperberg [21] who proposed the following simple single-qubit evolution that can be seen as a counterexample:

"Suppose that you have one qubit. Let $X(\theta) = \exp(iX\theta)$, and let $Z(\theta) = \exp(iZ\theta)$, where X and Z with no arguments are Pauli matrices. Let ϵ and δ be small numbers. You apply $X(\epsilon)$ and I apply $Z(\pm\delta)$ and we alternate. I choose between $Z(\delta)$ and $Z(-\delta)$ randomly. Let

$$E(\rho) = (Z(\delta)\rho Z(-\delta) + Z(-\delta)\rho Z(\delta))/2 \tag{11}$$

So the evolution is

$$X(\epsilon) \cdot Z(\pm\delta) \cdot X(\epsilon) \cdot Z(\pm\delta) \cdot X(\epsilon) \cdots \tag{12}$$

Now, if I tell you which operators I picked, this entire evolution is unitary and there is no noise. If I do not tell you, then the evolution is

$$X(\epsilon) \cdot E \cdot X(\epsilon) \cdot E \cdots , \tag{13}$$

where this new equation is at the level of operations and not operators. The point is that these two evolutions are consistent with each other. (13) is the average of (12) with respect to the classical choices that I make, so there is no contradiction between my telling you or not telling you. Also, if $1 >> \delta >> \epsilon$, (13) is an interesting evolution that reduces the qubit to a bit, but does not destroy everything. The bit survives in the so-called 'adiabatic limit,' even if the total application of X is many full rotations.

On the other hand, if you apply relation (3) of (13), then it is not the adiabatic limit. If the convolution is spread across a full rotation in the X direction, then the average in your formula becomes full depolarization."

7.3 Evolution of single qubits ion trap computers

Can long and complicated single-qubit evolutions be implemented with very little noise? Kuperberg's example above is based on a long alternating product of X and Z rotations by small angles. In the example, the X rotation is always in the same direction; each Z rotation is a random choice between the two directions to rotate. To be specific we asked about a product of 100 X rotations by +10 degrees and 100 Z rotations by ± 20 degrees that still has a fidelity of 50%. It turns out that the answer is yes.[4]

Realization by logical gates. Suppose that we want to demonstrate a certain quantum evolution on a single qubit. Rather than applying physically the gate to the qubit we regard the gate as changing the reference frame for the qubit and maintain on our classic computer the cumulative effect of the gates. Once we need to measure the state of the qubit we take into account the effect of the gates that we had to operate. In this way we "implement" an arbitrary evolution and the amount of noise depends only on our ability to perform the final measurement. (For various purposes it can be best to perform the X-operations physically and the Z-operations logically.)

Realization by "physical" gates. It turns out that physically manipulating single qubit ion trap is fairly robust and Greg's suggestion is quite realistic. This requires having qubits that are not stable but rather with internal Z cycle. Note, however, that the low amount of noise takes into account also deviations of the qubit Z-cycle from a highly precise clock. So it seems that Kuperberg's example can be implemented even if we insist on "physical" rather than logical or partially logical implementations.

The ability to implement (even physically) almost noiselessly arbitrary evolutions on single qubit ion trap devices is not in direct conflict with our conjectures, but it does nevertheless, through Kuperberg's example, weaken the overall case for using smoothed Lindblad evolutions to describe quantum processes that do not enact QFT. One possible response is that smoothed Lindblad evolutions describe quantum evolutions up to "removable noise." (In this case, the noise created by keeping the random Z-rotations secret. Compare [15], Section 5.1.) But putting this suggestion on formal grounds is not going to be easy. Another possible response is that natural noisy quantum evolutions that cannot be described by smoothed Lindblad evolutions rapidly reduce to classical evolutions. Almost noiseless single-qubit evolutions are also in tension with Conjecture E regarding the rate of noise. A possible solution is having a measure of noncommutativity in Conjecture E which scales up exponentially with the number k of qubits for general quantum evolutions described by quantum computers with k qubits.

7.4 Basing noisy models of quantum computers on smoothed Lindblad evolutions

It would be interesting to know whether imposing relation (3) on standard models of noisy quantum computers allows us to derive Conjectures A and B and and to preclude quantum fault tolerance. The simplest version of this suggestion would be to check if the main conjecture from Section 2 implies Conjectures A and B from Section 4. Another interesting question is whether, for reversible noisy computation, imposing smoothing-in-time through relation (3) reduces the computational power to BPP. (Without smoothing-in-time it is possible that log-depth quantum circuits prevail.) A harder task (which poses several technical and conceptual difficulties even to formulate,) would be to study the situation when we take noiseless classical computation for granted and continue to assume an unlimited supply of "cold ancillas." (We should get both these assumptions for free from Conjecture E on the rate of noise.)

7.5 Drunken time and drunken computation

One interpretation of relation (3) is that the leaks of information from an open quantum system to the environment cannot be properly timed. Noise accumulation can be regarded as asserting that the time flow of the environment, seen from inside the system, is stochastic. Pushing this idea one step further, we may consider stochastic-time smoothing of quantum processes without making a distinction between the intended evolution and the noise. While such "drunken-time" processes are further away from the issue of controlled quantum systems and noisy quantum computers it will be interesting to explore them. Indeed, as Lindblad evolutions (and more gen-

[4] I am thankful to Manny Knill and David Wineland for their kind detailed explanations.

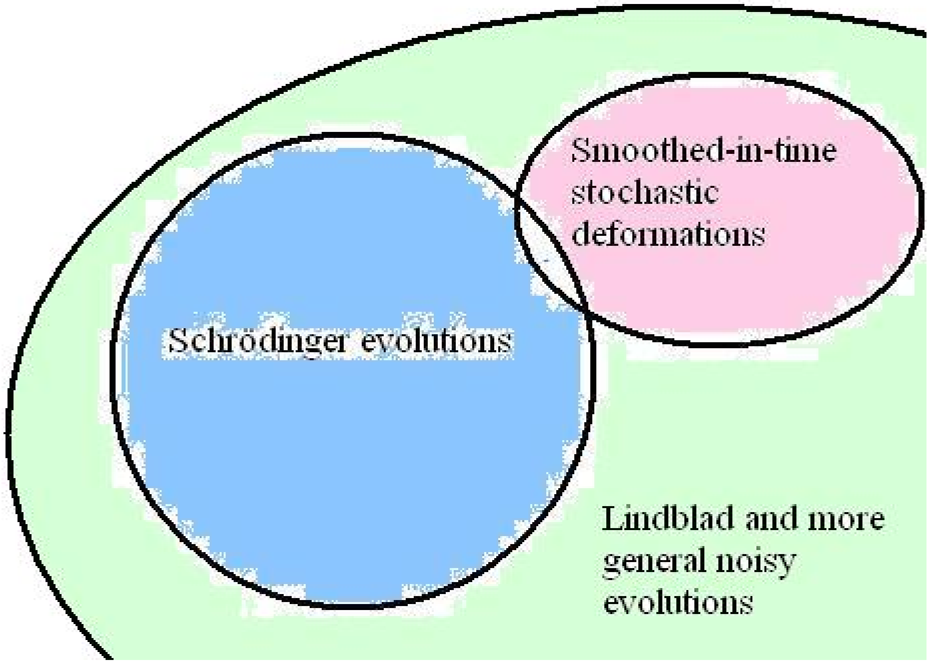

Figure 3. It is an appealing idea to describe decoherence as (yet-to-be-defined) time-stochastic deformation of Schrödinger evolutions

eral types of noisy evolutions) are all driven by Schrödinger equations (and can be expressed by unitary evolutions on a larger space) it will be of interest to express smoothing-in-time based just on Schrödinger equations.

It is an interesting thought that time flow has a stochastic nature which intensifies in quantum evolutions as we move away from classical evolutions. It is an appealing possibility that the behavior of decoherence can be modeled based on such stochastic-time evolutions. (See Figure 3.)

Also in computational complexity (both for classical and quantum computation) adding stochastic ingredients to the "flow of time" is of interest. See [16] for further (informal) discussion.

8 Quantum computers, noise, fault tolerance, and the threshold theorem

8.1 Quantum computers and noisy quantum computers

This Section provides background on the models of quantum computers and noisy quantum computers. We assume the standard model of quantum computer based on qubits and gates with pure-state evolution. The state of a quantum computer with n qubits is a unit vector in a complex Hilbert space $\mathcal{H}$: the 2^n-dimensional tensor product of 2-dimensional complex vector spaces for the individual qubits. The evolution of the quantum computer is via "gates." Each gate g operates on k qubits, and we can assume $k \leq 2$. Every such gate represents a unitary operator on the (2^k-dimensional) tensor product of the spaces that correspond to these k qubits. At every "cycle time" a large number of gates acting on disjoint sets of qubits operate. We will assume that measurement of qubits that amount to a sampling of 0-1 strings according to the distribution that these qubits represent is the final step of the computation.

The basic locality conditions for noisy quantum computers asserts that the way in which the state of the computer changes between computer steps is approximately statistically independent for different qubits. We will refer to such changes as “storage errors” or “qubit errors.” In addition, the gates that carry the computation itself are imperfect. We can suppose that every such gate involves a small number of qubits and that the gate’s imperfection can take an arbitrary form, and hence the errors (referred to as “gate errors”) created on the few qubits involved in a gate can be statistically dependent. We will denote as “fresh errors” the storage errors and gate errors in one computer cycle. Of course, qubit errors and gate errors propagate along the computation. The “overall error” describing the gap between the intended state of the computer and its noisy state takes into account also the cumulated effect of errors from earlier computer cycles.

The basic picture we have of a noisy computer is that at any time during the computation we can approximate the state of each qubit only up to some small error term ϵ. Nevertheless, under the assumptions concerning the errors mentioned above, computation is possible. The noisy physical qubits allow the introduction of logical “protected” qubits that are essentially noiseless.

In this paper we will consider the same model of quantum computers with more general notions of errors. We will study more general models for the fresh errors. (We will not distinguish between the different components of fresh errors, gate errors and storage errors.) Our models require that the storage errors not be statistically independent (on the contrary, they should be very dependent) or that the gate errors not be restricted to the qubits involved in the gates and be of sufficiently general form.

There are several other models of quantum computers that are equivalent in terms of their computational power to the one described here. This equivalence does not extend automatically to noisy versions and exploring fault tolerance in noisy versions of these models is an important challenge in FTQC.

8.2 The threshold theorem

We will not specify the noise at each computer cycle but rather consider a large set, referred to as the *noise envelope*, of quantum operations the noise can be selected from.

Let $\mathcal{D}$ be the following envelope of noise operations for the fresh errors: the envelope for storage errors $\mathcal{D}_s$ will consist of quantum operations that have a tensor product structure over the individual qubits. The envelope for gate errors $\mathcal{D}_g$ will consist of quantum operations that have a tensor product structure over all the gates involved in a single computer cycle (more precisely, over the Hilbert spaces representing the qubits in the gates). For a specific gate the noise can be an arbitrary quantum operation on the space representing the qubits involved in the gate. (The threshold theorem concerns a specific universal set of gates $\mathcal{G}$ that is different in different versions of the theorem.)

Theorem 8.1 (Threshold theorem)**.** *[2, 18, 19] Consider quantum circuits with a universal set of gates $\mathcal{G}$. A noisy quantum circuit with a set of gates $\mathcal{G}$ and noise envelopes $\mathcal{D}_s$ and $\mathcal{D}_g$ is capable of effectively simulating an arbitrary noiseless quantum circuit, provided that the error rate for every computer cycle is below a certain threshold $\eta > 0$.*

The value of the threshold in original proofs of the threshold theorem was around $\eta = 10^{-6}$ and it has since been improved by at least one order of magnitude. Recently, Knill [20] used error-detection codes rather than error-correction codes and massive post-selection for raising the value of η (based on numerical simulations) to 3%. (It also leads to substantially higher provable bounds [7].)

The threshold theorem relies on another important assumption. It is allowed to add new qubits, “cold ancillas” that are initialized to an error-free state $|0\rangle$. Roughly speaking, they are needed to “cool” the system. We will continue to make this assumption for our adversarial noise models throughout the paper.

The threshold theorem and some of its recent versions give a fairly good description of the board models of noise that allow universal quantum computing when the noise rate is sufficiently small. There are several results ([4, 27, 17]) showing that for the standard noise models when the computation is reversible or when the noise rate is high, the computational power reduces to BPP (for some results) or BPP^{BQNC} (the power of classical computers together with log-depth quantum circuits).

9 More on correlated errors

9.1 Generic noise

Proposition 9.1. *Conditioning on the expected number αn of qubit errors, a random unitary operator acting on all the qubits of the computer yields a very strong error synchronization.*

The proposition extends to the case where we allow additional qubits representing the environment. The proof of Proposition 9.1 is based on a standard "concentration of measure" argument (see, e.g., [24]). (We will give only a rough sketch.) When we consider a typical expression of the form $\sum a_w P_w$ where $\sum a_w^2 = 1$ and $\sum \{a_w^2 |w|\} = an$, it will have a large support on $a_{\mathbf{0}}$ and the other coefficients will be supported on a_w where w itself is typical; i.e., I (the error syndrome) behaves like a random string of length n with entries I,X,Y,Z. Hence $|w|$ is around $(3/4)n$.

How relevant is Proposition 9.1? It is well known that random unitary operations on the entire 2^n-dimensional vector space describing the state of the computer are not "realistic" (in other words, not "physical" or not "local"). The best formal explanation why random unitary operators are "not physical" is actually computational and relies on the following well-known

Proposition 9.2. *For large n, it is impossible to express or even to approximate a random unitary operator using a polynomial-size quantum circuit with gates of bounded fan-in (namely, gates that operate on a bounded number of qubits).*

9.2 The boundary of the threshold theorem

Recent works [30, 7, 3] show that the threshold theorem prevails if we allow certain space- and time-dependencies for the noise operations. We would now like to draw a distinction between noise models that support the threshold theorem and noise models that do not. For a quantum operation E describing the noise for a quantum computer with n qubits we denote by $\alpha(E)$ the expected number of qubit errors in terms of the multi-Pauli expansion as described above.

Proposition 9.3. *For the known noise models that allow FTQC via the threshold theorem:*

1) The fresh noise E expanded in terms of multi-Pauli operations decays exponentially above $\alpha(E)$.

2) The overall (cumulated) noise E' expanded in terms of multi-Pauli operations decays exponentially above $\alpha(E')$.

There is an even simpler property of fresh and cumulated noise for noise models for which the threshold theorem holds.

Proposition 9.4. *For the known noise models that allow FTQC via the threshold theorem:*

3) The fresh noise (at every computer cycle) for almost every pair of qubits in the computer is almost statistically independent for the two qubits in the pair.

4) The overall noise for almost every pair of qubits in the computer is almost statistically independent for the two qubits in the pair.

Here when we talk about "almost every pair" we refer to $(1 - o(1))\binom{n}{2}$ of the pairs when n is large.

The (rich) error syndrome will provide a simple way to express correlation between the noise acting on two qubits. For two qubits i and j, denote by $cor_{ij}(E)$ the correlation between the events that the qubit i is faulty and the event that the qubit j is faulty. In other words, $cor_{ij}(E)$ is the correlation between the events that w_i is not I, and w_j is not I when w is a word drawn according to the distribution of error syndromes described by E.

Proposition 9.4 implies, in particular, that for models allowing the threshold theorem, $cor_{ij}(E)$ and $cor_{ij}(E')$ are close to 0 for most pairs i, j of qubits. We will further discuss two-qubit behavior in Section 4.

Note that properties 1 and 3 refer to the noise model, which is one of the assumptions for the threshold theorem, while properties 2 and 4 are consequences of the threshold theorem and, in particular, of suppressing error propagation. For the very basic noise models where the storage errors are statistically independent property 3 follows from the fact that the number of pairs of interacting qubits at each computer cycle is at most linear in n. Property 3 continues to hold for models that allow decay of correlations between qubit errors that depend on the (geometric) distance between them. Property 1 is a simple consequence of the independence (or locality) assumptions on the noise for noise models that allow the threshold theorem.

10 Response to some critiques

> *"While it is true that quantum error correction seems necessary for building a general purpose quantum computer, it is not at all clear that un-error-corrected quantum systems are easy to efficiently simulate. For example, no one currently can efficiently simulate large condensed matter systems, such as those arising in high temperature superconductors, on a classical computer. But it is unlikely that these systems enact error correction."*

A main point of this paper is to offer a formal definition for the phrase "it is unlikely that these systems enact error correction." A main goal is to mathematically describe systems that do not enact quantum error correction and we propose to describe such systems by smoothed Lindblad evolutions.

As for modeling and simulations, there are many real-life processes that we do not know how to model and that we cannot simulate. The question is whether the current difficulty in simulating high temperature superconducting reflects the fact that describing this process requires superior computational power. This is an exciting possibility but we have no evidence that this is the case. It is more likely that we simply do not understand high temperature superconducting well enough. If there indeed are "un-error-corrected" systems that demonstrate superior computational power (namely, that cannot be simulated on classical computers because of their limited computational power) this would be very interesting as well. It is an important problem if we can achieve superior computational power on noisy quantum devices without using full-fledged FTQC.

> *"The author seems to imply that highly entangled quantum states have not been observed in nature. This is certainly not the case."*

Very, very highly entangled states of the kind used in quantum error correction and quantum algorithms have not been observed in nature. Conjecture C is an attempt to formalize this point.

> *Referring to Conjectures A and B: "The author should CLEARLY state how such an error model would work. I do not believe that it is possible to derive such a model without violating locality or the linearity of quantum theory."*

Let us consider a very special case of Conjectures A and B just for ion trap computers based on current architecture. It may be possible that if we can lower the noise level (of the individual qubits and gates) below the threshold then such computers can already serve as universal quantum computers. However, one concern is that for this particular architecture, moving the ion traps to cause them to interact will lead to noise synchronization. This can be examined experimentally or by describing a more detailed model that takes the specific architecture into account. This concern for this specific architecture does not violate locality or the linearity of quantum theory.

Now, for the general case, indeed Conjectures A and B are much bolder. They assert that *every* implementation of noisy quantum computers leads to noise synchronization for highly entangled states. The clear description of "how the model works" asked by the reader may well depend on the specific implementation. The idea behind our Conjectures A and B is to try to identify properties that hold universally while the specific reasons for "how the noise model works" are external to the proposed abstract model of noisy quantum computers and depend on the implementation.[5]

> *"Ultimately, the question is what Hamiltonian dynamics is realized in physics. For example, no physicist would believe that Hamiltonians with n-body interaction are naturally occurring. The reason is that most degrees of freedom are physically local and hence there is no way n distinct objects in n distinct locations can interact simultaneously without violating some notion of causality. Similarly, also on the basis of locality and causality one expects that a few body interactions which exist are between states that are physically nearby. Error synchronization is easily achieved if one assumes many body interactions or long-range interactions but the author should try to link his mathematical models to physical models."*

This comment indeed touches on the crux of matters and I have a few remarks to make. The first remark is that I believe that we do witness error synchronization in natural quantum processes. If you wish, for example, to create a specific bosonic state based on Bose-Einstein condensation on a bunch of very cold atoms[6] then mixture with undesirable bosonic states will amount to error synchronization on the "atomic basis." Second, it is correct that assuming that "fresh noise" for certain noisy quantum computers at certain states exhibits error synchronization may be unphysical and goes against the beliefs of physicists. What we suggest is that this occurs for cases that the states themselves are unphysical. (See Figure 2.) Finally, the thought that there is no way n distinct objects in n distinct locations can interact simultaneously without violating some notion of causality is already in tension with the type of interactions that give quantum computers superior computational power. Should we believe that complicated long-range Hamiltonians with n-body interactions can be achieved by quantum computers? Indeed, there is a compelling argument that quantum error correction will allow the creation of such complicated interactions. However, such developments represent an uncharted territory, and so does the appropriate relevant way to model noise.

[5]The following analogy can be of some help. The widely believed conjecture that $NP \neq P$ asserts that there is no efficient algorithm for solving large 3-SAT problems. The conjecture does not give a clear explanation why certain algorithms, classes of algorithms or hypothetical strategies to solve the problem will fail.

[6]From the time the theoretical framework was laid, it took many decades and quite a few scientific and technological breakthroughs, until Bose-Einstein condensation was experimentally demonstrated. As such, this story is a great "role model" for the quantum computer endeavor.