When Noise Accumulates - How Quantum Error Correction Might Fail

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Are quantum computers possible?

The feasibility of computationally superior quantum computers is one of the most fascinating and clear-cut scientific problems of our time.

The main concern regarding quantum-computer feasibility is that quantum systems are inherently noisy. This concern was put forward in the mid-90s by Landauer, Unruh, and others. (Better understanding of decoherence is a problem of great importance.)

Quantum error correction and FTQC

The theory of quantum error correction and fault-tolerant quantum computation (FTQC) and, in particular, the *threshold theorem*, which asserts that under certain conditions FTQC is possible, provide strong support for the possibility of building quantum computers.

There is surprisingly little effort devoted these days to trying to pursue a negative answer. Robert Alicki's works (some with various coauthors) are a notable exception.

The motivating reason for quantum computers

The rationale behind computationally superior quantum computers (Feynman, others): Digital computers are incapable of making certain computations from quantum physics; those require exponential time on digital computers.

However, the highly entangled states that demonstrate the computational superiority of QC and that are needed for quantum fault tolerance are not encountered in nature. This observation somewhat weakens the motivating rationale.

QEC and FTQC: Hope and concern

The hope: No matter what the quantum computer computes or simulates, nearly all of the noise will be a mixture of states that are not codewords in the error correcting code, but which are correctable to states in the code.

The concern: The process for creating a quantum error correcting code will necessarily lead to a mixture of the desired codeword with undesired codewords.

This lecture

The lecture comprises three parts:

1. What could serve as an alternative model of noisy quantum computers?
2. What can we say about general noisy quantum systems? How can we model quantum evolutions that do not enact quantum error correction?
3. Physics.

Part I: Noisy quantum computers

The computational model:

- Universal quantum computer with unitary gates.
- Noise envelope: a set of quantum operations representing the (fresh) noise at a single computer cycle. (These are further divided into storage noise and gate noise.)

What can cause quantum computers to fail?

1. We cannot go below the threshold.
2. The rate of noise scales up noticeably with the number of qubits.
3. The errors are badly correlated.

Correlation and rate

Observation: When the noise is very correlated there is a discrepancy between measuring errors in terms of individual qubits and measuring errors in terms of trace distance for the entire state. For highly correlated errors, if we fix the trace distance (for tiny time-intervals) as the measure, the number of qubit-errors scales up.

Systematic relation between the noise and the state

An important point: it is possible (in my opinion) that there is a systematic relation between the noise and the intended state of a noisy quantum computer. Such a systematic relation does not the violate linearity of quantum mechanics, and it is expected to occur in processes that do not exhibit fault tolerance.

Conjecture B: Error synchronization

In any noisy quantum computer in a highly entangled state there will be a strong effect of error synchronization.

(This is a physics conjecture.)

Conjecture A: Two qubits behavior

A noisy quantum computer is subject to error with the property that information leaks for two substantially entangled qubits have a substantial positive correlation.

(This is a conjecture about appropriate modeling of noisy quantum computation.)

How to express these conjectures mathematically and what are the consequences

I found that the best way to express error-synchronization (Conj. B) and positive correlation for information leaks (Conj. A) is by the expansion to product of Pauli operators.

We need a stronger form of Conjecture A where "entanglement" is replaced by a measure of expected entanglement based on (separably) measuring the other qubits in an arbitrary way.

Conjecture C: Statistical censorship

"Very highly entangled states" are completely infeasible for noisy quantum computers.

Part II: How to model un-suppressed noise accumulation?

Smoothed-in-time Lindblad evolution

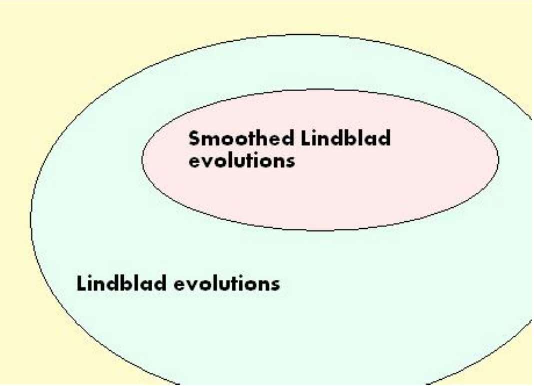

Figure: 1. Smoothed Lindblad evolutions are a restricted subclass of the class of all Lindblad evolutions

Smoothed (conjugated) Lindblad evolution

We start with a unitary evolution at time-interval [0,1]. $U_{s,t}$ is a unitary operator describing the change from time s to time t.

Next we consider a general Lindblad evolution obtained by adding noise expressed infinitesimally at time t by E_t.

We replace E_t by the weighted average of $U_{s,t} E_s U_{s,t}^{-1}$ over all times s with respect to a kernel $K(t-s)$.

Smoothed-in-time Schrödinger evolutions

It is possible that similar properties can be obtained by starting with Schrödinger evolutions and applying (more standard) smoothing-in-time.

Smoothed-in-time Schrödinger evolutions

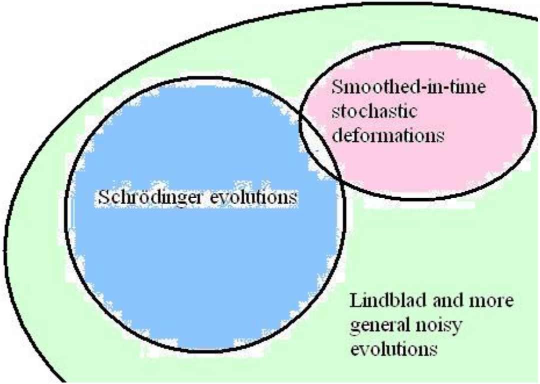

Figure: 2. It is an appealing idea to describe decoherence as (yet-to-be-defined) time-stochastic deformation of Schrödinger evolutions

The rate of decoherence

A satisfactory description of decoherence should propose a lower bound for the rate of noise. Here is an informal suggestion.

A noisy quantum system is subject to noise whose rate at a time-interval $[s, t]$ (in terms of trace distance) is bounded from below by a measure of noncommutativity for the set of operators describing the evolution in this interval.

Is time stochastic?

Is time flow stochastic?

Because of the Law of Large Numbers, small-scale stochastic processes look completely deterministic in larger scales. So if time flow is stochastic in some scales we may not notice it in larger scales. (However, if time flow is stochastic, I would expect the variance to express some measure of the non-classical behavior of the dynamics.) Thinking of time as stochastic may be a mathematically useful device.

Two further conceptual questions

1. How is it possible that quantum error correction fails while classical error correction prevails?

2. If we know that a controlled noisy quantum evolution is near the intended evolution at all times, could there be systematic relations between the noise at one time and the evolution at a later time? (Proposed answer: yes.)

Physics: Are our conjectures unphysical?

Several people have commented that our suggested properties of noise for some (hypothetical) quantum computer architecture at some quantum state ρ allow instantaneous signaling, and thus violate basic physical principles. That is entirely correct, but our proposed conclusion is that this quantum computer architecture simply does not accommodate the quantum state ρ.

This response applies to various other concerns of a similar nature.

Physics: Are our conjectures unphysical?

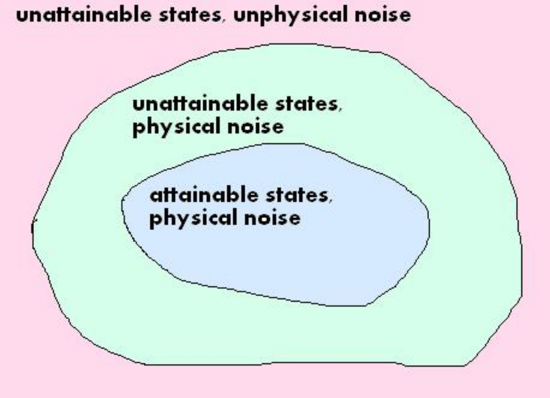

Figure: 3. Given a proposed architecture for a quantum computer it is possible that for some hypothetical states that cannot be achieved the proposed properties of noise are "unphysical." The place to examine the conjectures is for attainable states.

Physics: Ion trap computers

One place to examine some suggestions of this lecture is current implementations of ion-trap computers. In these implementations we need to move qubits together in order to gate them, and this suggests that, in each computer cycle, errors will be correlated for *all* pairs of qubits.
At present, the rate of noise is still the major concern of experimentalists, but it is not clear how a large pairwise correlation between all pairs of qubits can be avoided in the current architecture. This is an example, where properties of accumulated noise occur for other reasons.

Physics: Bosonic states

Consider simulating bosonic states with a noisy quantum computer. When errors accumulate I expect that a large (even dominant) part of the noise will not consist of local noise based on the computational bases but rather it will be a mix of the intended bosonic state with other unintended bosonic states.
Noise accumulation seems consistent with the familiar property of physical systems where the low-scale structure is not witnessed when we look at larger scales.

Physics: Bose-Einstein condensation

We do not yet have quantum computers that simulate bosonic states but we do have several natural and experimental processes that come close to this description. Consider Bose-Einstein condensation on cold atoms. Describing the bosonic state in terms of individual atoms is analogous to describing a complicated state of a quantum computer in terms of the computational basis. This analogy enables us to ask if the deviation of a state created experimentally from a pure state can be described by independent noise operators on the different atoms.

We propose a different picture, namely, that a state created experimentally can be described as a mixture of different pure Bose-Einstein states and that, with respect to any pure state it approximates, it exhibits strong error synchronizations on the "atomic basis" for the underlying Hilbert space.

Physics: Anyons and topological quantum computing

Another comment was that FTQC via topological quantum computing does not rely on the threshold theorem and Conjectures A and B are not relevant for this model.

However, the extreme stability to noise expected for anyonic systems relies on similar assumptions to those enabling quantum error correction.

Physics: Noisy anyonic systems

Creating anyonic systems that may allow FTQC via topological quantum computing already requires fault-tolerant processes. When we create Abelian anyons in the laboratory, or try to create non-Abelian anyons, there is no reason to believe that the process for creating them will involve suppression of propagated noise. I conjecture that smoothed Lindblad evolutions describe the current and proposed processes for creating anyons, and that just as when we simulate fermions or bosons, we will witness a mixture of the intended state with other states of the same type and we will not witness the strong stability of certain anyonic systems that is predicted by current models. Of course, these matters are subject to intense experimental examinations.

Physicist:

Here is an experimental process for creating non-Abelian anyons that can be useful for universal quantum computing.

Computer scientist: If the process itself does not involve fault tolerance then the anyons will not be useful.

Physicist: Can you prove this?

Mathematician (optimistically): It will take me a decade to formally define the statement (if it is possible) and another decade to supply a proof (if it is true).

Physicist: I will create the anyons sooner.

Conclusion

The laws for quantum processes which do not enact quantum error correction are interesting.

Understanding such systems may be relevant to various physical questions, including the question whether quantum computers are possible.